Valuation of Environmental Costs and Benefits

Zhenting Wang, Zijian Xing, Huatong Huang, Shengxiang Zhang[a, *]

College of Mathematics and Informatics, South China Agricultural University, Guangzhou, China

[a]sxzhang@scau.edu.cn

*Corresponding author

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Abstract: Economic theory often disregards the impact of its decisions on the biosphere. So we hope to create an ecological service, then the true and comprehensive valuation of the project can be determined and assessed. We use Grey Prediction and Principal Component Analysis (PCA) to create an ecological services valuation model which matches reality and need to change over time. Then we choose the Three Gorges Dam of the Yangtze River in China for research. According to the results, the environmental governance of the Yangtze River started late, so the benefits of certain environmental governance will be very obvious. Therefore, we should pay more attention to ecological protection while strengthening economic construction.

1. Introduction

Previously, people pursue short-term economic interests blindly, which results in serious damage to the environment. But the ecosystem services are not free [1]. The unreasonable land use is a very important reason for environmental degeneration [2]. In this paper, we will create an ecological services valuation model.

2. Assumptions and Justification

Assume that there are no major natural disasters.

Assume that the environment of the project has a certain tolerance and self-repair ability

Assume that the unit price of pollutant treatment that we estimate is basically realistic.

3. The establishment and solution of models

3.1 Background

For different sizes of land use, there will be differences in types and quantities of evaluation indicators. When we need to assess the environmental cost of land use development projects, we need to collect a number of data and use them to analyze and find the law. We need to collect a number of data and use this observational data with multiple variables to analyze and find the law. We need to find a method to minimize the loss of information contained in the original indicators while reducing the indicators that need to be analyzed [3]. Therefore, we use PCA which is a kind of dimension reduction algorithm [5] to solve the problem.

The main idea of PCA is to map n dimension to k dimension. According to the knowledge of probability theory, the closer the variance is to zero, the less discrete the data is [6]. To achieve the goal of dimensionality reduction, we need to find a new dimension with large variance to contain most of the original information. The work of PCA is to find a set of orthogonal coordinate axes from the original space sequentially.

For land use and development projects, the environment of the project will change and cost will be generated by time. Therefore, we need to update the evaluation model and predict the new cost and benefits of land use and development projects. And then we combined it with the principal components calculated by PCA to predict the new unknown principal components. Thus, we can predict the new cost and benefits of land use development projects through the Grey Prediction model.

The Grey Prediction is developed on the basis of ordinary differential theory [5]. Part of the information in the grey system is known. There are uncertain relationships among the factors in the system. We identify the degree of difference between the development trends of system factors, and then generate and process the original data to find the rules of system changes, generate an ordered sequence of data. Then we build the corresponding differential equation model to predict the future development trend of things [6].

Through the PCA and the Grey Prediction, we can get cost estimates of land of different sizes. Then we use GDP to reflect benefits, we can get cost-benefit ratio.

3.2 Implementation steps

We assume that the cost index observation data matrix is:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$$

1) Standardize the raw data:

$$x_{ij}^{*} = \frac{x_{ij} - \overline{x}_j}{\sqrt{Var\left(x_j\right)}} \left(i=1,2,\cdots,n;\ j=1,2,\cdots,\mathrm{p}\right) \tag{1}$$

$$\left(\overline{x}_j = \frac{1}{n}\sum_{i=1}^{n} x_{ij}, Var\left(x_j\right) = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij} - \overline{x}_j\right)^2 \left(j=1,2,\cdots,p\right) \right) \tag{2}$$

2) Calculate the sample correlation coefficient matrix:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1p} \\ r_{21} & r_{22} & \cdots & r_{2p} \\ \vdots & \vdots & & \vdots \\ r_{p1} & r_{p2} & \cdots & r_{pp} \end{bmatrix}$$

$$\left(rij = \frac{Cov(xi, xj)}{\sqrt{Var(xi)}\sqrt{Var(xj)}} = \frac{\sum_{k=1}^{n}(xki - \overline{xi})(xkj - \overline{xj})}{\sqrt{\sum_{k=1}^{n}(xki - \overline{xi})^2}\sqrt{\sum_{k=1}^{n}(xkj - \overline{xj})^2}} \right) \tag{3}$$

3) Calculate the eigenvalues and eigenvectors of the correlation matrix R:
Eigenvalues:

$$(\lambda_1, \lambda_2, \cdots, \lambda_p)$$

Eigenvectors:

$$\alpha_i = (\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{ip}) \quad (i = 1, 2, \cdots, p)$$

4) Choose the principal component and write the principal component expression:
PCA can get p principal component, so we need to select k principal component according to the contribution rate. Generally, the cumulative contribution rate is more than 85%. Among them, the contribution rate is the proportion of variance of a principal component to total variance. The expression is:

$$Contribution = \frac{\lambda i}{\sum_{i=1}^{p} \lambda i} \tag{4}$$

5) Calculate Principal Component Load:
The principal component load reflects the degree of correlation between the principal component Fi and the original variable Xj. The original variable Xj (j = 1, 2,..., p) is the load lij (i = 1, 2,..., m; J = 1, 2,..., p) on the principal component Fi (i = 1, 2,..., m).
6) Calculate principal component scores:
According to the standardized raw data, the principal component expression was introduced to obtain the principal component score of each sample. The specific form is:

$$\begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1k} \\ F_{21} & F_{22} & \cdots & F_{2k} \\ \vdots & \vdots & & \vdots \\ F_{n1} & F_{n2} & \cdots & F_{nk} \end{bmatrix}$$

$$\left(F_{ij} = a_{j1}x_{i1} + a_{j2}x_{i2} + \cdots + a_{jp}x_{ip} \quad (i = 1, 2, \cdots, n; j = 1, 2, \cdots, \text{k}) \right)$$

7) Determine the weight:
The principal component value of each variable X_i in the component matrix is divided by the square root of the corresponding eigenvalue $\sqrt{Y_i}$. And we can get the coefficient in the linear combination Z_{ji}

$$Z_{ji} = \frac{X_i}{\sqrt{Y_j}} (i = 1, 2, \cdots, 7; j = 1, 2) \tag{5}$$

We multiply the coefficient Z_{ji} in the linear combination by the corresponding variance contribution rate P_j. Then we sum it up, divide it by the sum of variance contribution rate, and get the coefficient in the comprehensive scoring model Q_i.

$$Q_i=\frac{\sum_{j=1}^{2}\left(Z_{ij}\times P_j\right)}{\sum_{j=1}^{2} P_j}(i=1,2,\cdots,7) \tag{6}$$

The coefficient Q_i in the comprehensive scoring model is normalized to get the weight R_i.

$$R_i=\frac{Q_i}{\sum_{i=1}^{7} Q_i}(i=1,2,\cdots,7) \tag{7}$$

8) Analysis and Modeling:

Follow-up analysis often includes comprehensive evaluation, regression analysis, and selection of variables subset and so on.

9) Accumulate data for Grey Prediction

In reality, data are often interfered by various aspects, including various noises. Accumulating data can not only play a role in the noise reduction, but also help to strengthen the exposition of mathematical laws. In this way, we can weaken the volatility and randomness of random sequence and get new data sequence.

We assume that there is an original sequence of numbers:

$$x^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right) \tag{8}$$

Now we add up $x^{(0)}$:

$$\begin{aligned} &x^{(1)}(1)=x^{(0)}(1)\\ &x^{(1)}(2)=x^{(0)}(1)+x^{(0)}(2)\\ &x^{(1)}(3)=x^{(0)}(1)+x^{(0)}(2)+x^{(0)}(3)\\ &\quad\vdots\\ &x^{(1)}(n)=x^{(0)}(1)+x^{(0)}(2)+x^{(0)}(3)+\cdots+x^{(0)}(n) \end{aligned} \tag{9}$$

We can get a new sequence:

$$x^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),x^{(1)}(3),\cdots,x^{(1)}(n)\right) \tag{10}$$

In the formula, $x^{(1)}(\mathrm{i})$ represents the superposition of the $x^{(0)}$ corresponding to the data of the preceding i:

$$x^{(1)}(i)=\sum_{j=1}^{i} x^{(0)}(j)\ \left(i=1,2,3,\cdots,n\right) \tag{11}$$

10) Calculate the grey differential equation of sequence $x^{(1)}$:

In order to restore the accumulated data to the original sequence, we need to do post-subtraction or symmetric subtraction. This refers to the difference between the last two data:

$$\Delta x^{(1)}(i)=x^{(1)}(i)-x^{(1)}(i-1)=x^{(0)}(i)$$

$$\left(i=1,2,\cdots,\mathrm{N},\ x^{(0)}(0)=0\right) \tag{12}$$

$$\left(\Delta x^{(1)}(i)=\frac{x^{(1)}(i)-x^{(1)}(i-1)}{i-(i-1)}\right)$$

This shows that there is a derivative relationship between the new sequence and the cumulative sequence. We assume that $x^{(1)}$ satisfy differential equation of first order:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=u \tag{13}$$

In the formula above, a is a constant. And u is the development of grey number, which is the endogenous control of gray level, in other words, a long-term constant input to the system. Then we use the general formula of ordinary differential to solve the grey differential equation:

$$x^{(1)}(t)=\left[x^{(1)}(t_0)-\frac{u}{a}\right]e^{-a(t-t_0)}+\frac{u}{a} \tag{14}$$

The discrete values of equal interval sampling are $(\mathrm{t}_0=1)$:

$$x(k+1)=\left[x^{(1)}(1)-\frac{u}{a}\right]e^{-ak}+\frac{u}{a} \tag{15}$$

Then we estimate constants a and u by least square method. First of all, $x^{(1)}$ will be used as the initial value, so $x^{(1)}(2), x^{(1)}(3), ..., x^{(1)}(\mathrm{N})$ will be substituted into grey differential equation separately. We use difference instead of differential. And because of the equal interval sampling, $\Delta t=(\mathrm{t}+1)-\mathrm{t}=1$, So we can get:

$$\frac{\Delta x^{(1)}(2)}{\Delta t}=\Delta x^{(1)}(2)=x^{(1)}(2)-x^{(1)}(1)=x^{(0)}(2) \tag{16}$$

$$\frac{\Delta x^{(1)}(3)}{\Delta t}=x^{(0)}(3),\cdots,\frac{\Delta x^{(1)}(N)}{\Delta t}=x^{(0)}(N) \tag{17}$$

Thus, because of the grey differential equation, we can get:

$$\begin{cases} x^{(0)}(2)+ax^{(1)}(2)=u \\ x^{(0)}(3)+ax^{(1)}(3)=u \\ \cdots\cdots\cdots \\ x^{(0)}(N)+ax^{(1)}(N)=u \end{cases} \tag{18}$$

Then we move $ax^{(1)}(\mathrm{i})$ to the right and write it in the form of a product of quantities inward:

$$\begin{cases} x^{(0)}(2)+ax^{(1)}(2)=u \\ x^{(0)}(3)+ax^{(1)}(3)=u \\ \cdots\cdots\cdots \\ x^{(0)}(N)+ax^{(1)}(N)=u \end{cases} \quad (19)$$

Because $\frac{\Delta x^{(1)}}{\Delta t}$ involve the value of two moments of the cumulative column $x^{(1)}$, it is more reasonable to take the average of the two moments before and after. So $x^{(i)}(i)$ will be replaced by:

$$\frac{1}{2}\left[x^{(i)}(i)+x^{(i)}(i-1)\right],(i=2,3,\cdots,N) \quad (20)$$

We rewrite the product of vectors to the expression of matrices:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ M \\ x^{(0)}(N) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(2)+x^{(1)}(1)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(3)+x^{(1)}(2)\right] & 1 \\ M & 1 \\ -\frac{1}{2}\left[x^{(1)}(N)+x^{(1)}(N-1)\right] & 1 \end{bmatrix} \quad (21)$$

We assume that:

$$y = \left(x^{(0)}(2),x^{(0)}(3),\cdots,x^{(1)}(N)\right)^T \quad (22)$$

So the matrix form is $y = BU$. The least squares estimates are as follows:

$$\hat{U}=\begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix}=\left(B^T B\right)^{-1} B^T y \quad (23)$$

We substitute the estimated values $\hat{a}$ and $\hat{u}$. And then we can get the time response equation:

$$\hat{x}^{(1)}(k+1)=\left[x^{(1)}(1)-\frac{\hat{u}}{\hat{a}}\right]e^{-\hat{a}k}+\frac{\hat{u}}{\hat{a}} \quad (24)$$

When $k = 1, 2, ..., N-1$, $\hat{x}^{(1)}(k+1)$ is the fitting value;

When $k \geq N$, $\hat{x}^{(1)}(k+1)$ is the predicted value. This is the fitting value equivalent to a cumulative sequence $x^{(1)}$. We restore it by subtraction. When $k = 1, 2, ..., N-1$, we can get the fitting value of the original sequence $x^{(0)}$ is $\hat{x}^{(0)}(k+1)$; when $k \geq N$, we can get the predicted value $x^{(0)}$ of the original sequence.

11) Collect data of benefits and use the Grey Prediction to get the predicted benefits. Through the PCA and the Grey Prediction, we can get cost-benefit ratio. Therefore, the true and comprehensive valuation of the project can be determined and assessed.

4. Study of The Three Gorges Dam Project

4.1 Background

We choose the Three Gorges Dam Project of the Yangtze River in China to use our model to perform a cost benefit analysis. We collected data of sewage, waste water, rubbish, pesticides, fertilizer, SO2 and NO2 from 2009 to 2017[4]. Then we can estimate the cost of each kind of environmental pollution. Then we use the Grey Prediction to get the budget value of each kind of environmental pollution cost in 2017. And then we use PCA to determine the weight, and finally use the accuracy test to get the prediction accuracy.

4.2 Prediction

We collected data of sewage, waste water, rubbish, pesticides, fertilizer, SO2 and NO2 from 2009 to 2017. Based on the data, we can get the cost estimates for each type of pollution. Then we use the Grey Prediction to get the predicted value of each kind of environmental pollution cost in 2017:

Table 1. 2017 Value of Environmental Pollution Cost Table

Sewage	30297297981
Waste Water	49607624289
Rubbish	9919436146
NO2	296004.6549
Pesticides	481596.2808
Fertilizer	73723707.98
SO2	98694.0328

4.3 Accuracy Test

We use the accuracy test of Grey Prediction. The posterior difference ratio C is 0.2119319 < 0.35, so the prediction accuracy is better.

4.4 Cost Estimation

Real cost estimation formula of the Three Gorges Dam:

$$S_1=0.688X_1-0.747X_2-0.703X_3+0.596X_4+0.823X_5+0.086X_6+0.258X_7 \tag{25}$$

The environmental cost of 2017 is about 2064820689 yuan by real data above.

Predicted cost formula of the Three Gorges Dam:

$$S_2=0.620X_1-0.688X_2-0.692X_3+0.379X_4+0.732X_5+0.208X_6+0.441X_7 \tag{26}$$

The environmental cost of 2017 is about 22155671229 yuan by real data above.

4.5 Cost-benefit ratio

Once ecosystem services are accounted for in the cost-benefit ratio of a project, then the true and comprehensive valuation of the project can be determined and assessed. Firstly, we collected GDP of the Three Gorges Dam. Secondly, we use the Grey Prediction and the data of 2009-2016 and we can get the Predicted Benefits is 8285.2301. Based on our model, we can get the value of the True Cost is 206.408 and the Predicted Cost is 221.557. At last, by dividing the real cost by the income ratio, we can get the Cost-Benefit Ratio, the True Cost-Benefit Ratio is 0.026593958 and the Predicted Cost-Benefit Ratio is 0.026741166. Therefore, the true and comprehensive valuation of the project can be determined and assessed.

5. Conclusion

Based on the above two case studies, we can know that the real values and predicted values of the Three Gorges Dam project's "ecological cost-benefit ratio" is 0.026593958 and 0.026741166. According to the "ecological cost-benefit ratio" and the actual situation, we can see that China started late in environmental governance, but the Yangtze River environmental governance has not achieved preliminary results. Therefore, investment in certain funds for environmental governance, the benefits will be very obvious. Besides, according to the results of precision test, Grey Prediction and PCA have better utility in estimating environmental costs and benefits. They are in line with actual cases.

6. Sensitivity Analysis

We must evaluate the accuracy of Grey Prediction for avoiding misuse of mathematical model. We adopt residual test, which is a point-by-point test of model accuracy and an intuitive arithmetic test. We also adopt post-residual test, which is carried out according to the probability distribution of residual and statistical test.

6.1 Implementation steps

6.1.1 Residual Test

We think that the general percentage $\pm 5\%$ is satisfactory, and the percentage $\pm 20\%$ can also be used according to the actual situation. If it is bigger, it is necessary to consider modifying the model or changing it to other models. In fact, if the original data swing is small, the accuracy is much smaller than $\pm 5\%$.

Table 2. Prediction Accuracy Rank Control Table

Prediction Accuracy Level	C
Good	<0.35
Qualified	<0.45
Reluctantly	<0.50
Unqualified	≥ 0.65

Then we need to calculate the following parameters separately:

Residual:

$$E(k)=x^{(0)}(k)-\hat{x}^{(0)}(k), k=2,3,\cdots,\mathrm{N} \tag{27}$$

Relative residual:

$$e(k)=\left[x^{(0)}(k)-\hat{x}^{(0)}(k)\right] / x^{(0)}(k), k=2,3,\cdots,\mathrm{N} \tag{28}$$

6.1.2 Post-residual Test

Post-residuals testing can be classified into four precision levels, as shown in the Table 1, then we need to calculate the following parameters separately:

Mean value of $x^{(0)}$:

$$\bar{X}=\frac{1}{N} \sum_{k=1}^{N} x^{(0)}(k) \tag{29}$$

Variance of $x^{(0)}$:

$$S_1=\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[x^{(0)}(k)-\bar{x}\right]^2} \tag{30}$$

Mean value of residual:

$$\bar{E}=\frac{1}{N-1}\sum_{k=2}^{N}E(k) \tag{31}$$

Variance of Residual Errors:

$$S_2=\sqrt{\frac{1}{N-1}\sum_{k=2}^{N}\left[E(k)-\bar{E}\right]^2} \tag{32}$$

Post-test difference ratio:

$$C=\frac{S_2}{S_1} \tag{33}$$

Therefore, we can know whether the model is qualified.

6.2 Test result

For the Three Gorges Dam Project of the Yangtze River in China, the posterior difference ratio C is 0.2119319 < 0.35, so this prediction accuracy is BETTER.

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